Electrical Re-entrainment of Particles in Wire-Duct Electrostatic Precipitators

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Abstract—This paper is aimed at investigating the re-entrainment phenomena of particles in wire-duct electrostatic precipitators. This calls for assessment of the forces acting on a particle positioned in the precipitator nearby the collecting plate. These include Van Der Waals, Johnsen Rahbek and repulsion forces. Van Der Waals and Johnsen Rahbek forces are directed toward the collecting plate and repulsion force is directed opposite toward the discharge wire. Re-entrainment takes place when the repulsion force exceeds the resultant of both the Van Der Waal and Johnsen Rahbek forces. To estimate the repulsion force, the electric field acting on the particle is required as calculated by using the charge simulation method. Conducting and dielectric particles are investigated, where the conducting particle is simulated by inner charges. However, the dielectric particle is simulated by inner and outer charges. Satisfaction of pertinent boundary conditions results in a set of equations whose solution determines the values of the simulation charges. With the determination of the simulation charges, the electric field and hence the repulsion force are evaluated. Re-entrainment takes place for dielectric particles at particle radius greater than 1 μ m in agreement with previous findings. The particle radius values at re-entrainment are smaller than those for metal ones.

Keywords—Electrostatic precipitator, particle re-entrainment, Van Deer Waal force, Johnsen Rahbek force, charge simulation method

I. INTRODUCTION

Electrostatic precipitators (ESPs) erected along the chimney of diesel engines suffer from particle reentrainment as this reflects itself on low efficiency of particle collection.

A DC energized ESP for treating the exhaust of a 1.7 kW diesel engine showed low efficiency due to particle re-entrainment. The worst efficiency reached -1200% for large particle size. Negative efficiency means that the outlet exhaust has a larger particulate mass than that of the inlet one. Particle size in the range of 300-1000 nm was excellent as regard to the efficiency (about 100% with no re-entrainment and no negative efficiency) [1].

A two-stage ESP was proposed [2] to treat the diesel engine exhaust. The precharge stage is aimed to charge the particles in a wire-plate geometry stressed positively. The collecting stage is aimed at collecting the particles in parallel-plate configuration. The collection efficiency for particles' diameter larger than 1 µm decreases with increasing the operating time until it reaches a constant value. The collection efficiency for particles' diameter larger than 2 µm is negative. The reason for this is attributed to the re-entrainment from collecting plate where collected particles are growing to larger ones, i.e.; agglomeration at the collecting plate. The larger the diameter of the particle the larger is the time for agglomeration to take place and the slower the collection efficiency to reach a constant value. This study is the only one to report also re-entrainment for particles less

than 0.5 μ m. This may be attributed to charging and collection of particles take place separately in different sections of the ESP [2].

In this paper, particle re-entrainment is investigated in a wire-duct ESP. This calls for assessment of the forces acting on a particle positioned in the precipitator nearby the collecting plate. These include Van Der Waals, Johnsen Rahbek and repulsion forces. Van Der Waals and Johnsen Rahbek forces are directed toward the collecting plate and repulsion force is directed opposite toward the discharge wire. Re-entrainment takes place when the repulsion force exceeds the resultant of both forces; Van Der Waal and Johnsen Rahbek. The repulsion force depends on the electric field and the others depend on the relative position of the particle with respect to the collecting plate and the discharge electrode. This calls for calculating the electric field where the particle is positioned in the vicinity of the collecting plate using the charge simulation technique. As the exhaust gases of diesel engine carry carbon particles, the electric field is calculated and hence the re-entrainment is assessed for both metal and dielectric particles with different values of particle radius and dielectric constant.

II. METHOD OF ANALYSIS

A. Charge Simulation Technique

A.1 Selection of Simulation Charges

In a wire-duct ESP stressed by voltage V, the wire has radius r_w and spacing H from the collecting plates as shown in Fig. 1. The surface charge on the discharge wire is simulated by N infinite line charges extending along the y-axis with x-z is the page plane. The charges

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Fig. 1. Discharge wire of the ESP extending along the y-axis.

are distributed equally around a cylinder with a radius $f_1.r_w$ where f_1 is a fraction. The *x*- and *z*-coordinates of the *j*th line charge, j = 1, 2, ... N are expressed as:

$$x_c(j) = f_1 \cdot r_w \cdot \sin(j \ast \Delta\theta) \tag{1-a}$$

$$z_c(j) = H + r_w + f_1 \cos(j * \Delta\theta)$$
(1-b)

where the angle $\Delta \theta \left(=\frac{2\pi}{N}\right)$ is as shown in Fig. 1.

A.1.1 Dielectric Particle

A dielectric particle of radius r_p and relative permittivity ε_r is positioned at height Z_o from the collecting plate. ε_r depends on the particle type. In case of dielectric particle, the particle is simulated by two sets of *n* ring charges, one inside and the other outside the particle [3, 4]. The inner charges are of radius β_1 . $r_p \sin \gamma$ where β_1 is a fraction. The outer charges are of radius $\beta_2.r_p \sin \gamma$, where β_2 is greater than 1. The angle γ is decided by the coordinates of the boundary point. The inner and outer ring charges are positioned at different *z*levels as shown in Fig. 1. Thus, the number of charges simulating the wire and the dielectric particle is N + 2n.

Images of the simulation charges of the discharge wire and the particle with respect to the grounded collecting plates are considered.

A.1.2 Metal Particle

In case of metal particle, the particle is simulated [3, 4] by a set of *n* point charges positioned inside it and distributed around the *z*-axis at varying *z*-level i.e.; over a zigzag path with a spacing β_3 . r_p from the particle center, where β_3 is a fraction. Thus, the total number of charges simulating the wire and the metal particle is N + n.

Images of the simulation charges of the discharge wire and the particle with respect to the grounded collecting plates are considered.



Fig. 2. Outer and inner simulation ring charges of a dielectric particle.

A.2 Boundary Point

To assess the values of the wire simulation charges, a set of *N* boundary points on the wire surface is selected.

The *x*- and *z*- coordinates of boundary points on the wire surface are expressed as:

$$x_b(i) = r_w \cdot \sin(i * \Delta \theta)$$
(2-a)
$$z_b(i) = H + r_w + r_{wire} \cdot \cos(i * \Delta \theta)$$
(2-b)

where x_b (*i*) and z_b (*i*) are *x*- and *z*- coordinates of the ith boundary point on the wire surface; *i* = 1, 2, ... *N*.

Each boundary point on the wire corresponds to a simulation line charge having the same radial direction from the wire axis as shown in Fig. 1.

To assess the values of charges simulating the particles, n boundary points are selected on the particle surface.

In case of dielectric particle, each boundary point on the particle surface corresponds to two simulation ring charges; one inside and other outside the particle. The boundary points on the dielectric particle are selected at the same *z*-level as that of the inner and outer ring charges, as shown in Fig. 2.

In case of metal particle, each boundary point corresponds to one simulation point charge. The boundary points on the metal particle are selected in the same radial direction from the particle center.

A.3 Boundary Conditions

A.3.1 Dielectric Particle

i) At the wire surface, one boundary condition is satisfied at each boundary point. The calculated potential due to the simulation wire charges, the inner simulation ring charges of the particle and their images are equal to the applied voltage *V*, which is the Dirichlet boundary condition. At the *i*th boundary point on the wire, the Dirichlet condition is satisfied as:

$$\sum_{j=1}^{N} P(i,j) \cdot q(j) + \sum_{\substack{j=N+1 \\ j=1,2, \dots, N+n}}^{N+n} P(i,j) \cdot q(j) = V,$$
(3-a)

where P(i, j) is the potential coefficient at *i*th boundary point due to *j*th charge and q(j) is the *j*th simulation charge of both the wire and the particle from inside.

- At the surface of the dielectric particle, two boundary conditions are satisfied at each boundary point:
 - i. The calculated potential is the same at the boundary point on the particle surface whatever the point is seen from the air outside the particle or from the dielectric inside the particle, and
 - ii. The continuity of the normal component of the electric flux in air and dielectric at any boundary point on the particle surface is to be maintained. This is Neuman boundary condition. The electric field at the particle surface when seen from the air side is equal to those calculated due to the simulation charges of the wire, the inner ring charges of the particle and their images. On the other hand, when the point is seen from the dielectric side, the calculation of electric field is due to the simulation charges of the wire, the outer ring charges of the particle and their images.

At the *i*th boundary point on the particle surface, the potential equality and the Neuman condition are satisfied:

$$\begin{split} \sum_{j=N+1}^{N+n} P(i,j). q(j) &= \sum_{j=N+n+1}^{N+2n} P(i,j). q(j) \quad (3-b) \\ \epsilon_r \left(\sum_{j=N+n+1}^{N+2n} E_n(i,j) + \sum_{j=1}^{N} E_n(i,j) \right) &= \\ \left(\sum_{j=N+1}^{N+n} E_n(i,j) + \sum_{j=1}^{N} E_n(i,j) \right) \quad (3-c) \end{split}$$

where $E_n(i, j)$ is the normal field coefficients at *i*th boundary point due to *j*th charge. These coefficients are expressed in Appendix.

A.3.2 Metal particle

At the wire and particle surfaces, one boundary condition is satisfied at each boundary point. The calculated potential due to the simulation wire charges, the inner simulation point charges of the particle and their images is equal to the applied voltage V, which is the Dirichlit boundary condition.

The particle will pick up an induced voltage which depends on its relative location with respect to the discharge wire. To access this induced potential, additional boundary condition is requested. Such boundary condition is expressed as the sum of the point charges simulating the particle is equal to zero [5].

A.4 Determination of Simulation Charges Values

Satisfaction of the above mentioned boundary condition at the boundary points selected at the wire and particle surfaces results in a set of equations whose simultaneous solution determines the unknown simulation line, ring or point charges.

To check the accuracy of the charge simulation procedure in satisfying the above-mentioned boundary conditions on the wire and particle surfaces, a set of *N* check points are chosen on the wire surface and another set of *n* check points are chosen on the particle surface. The check points are chosen midway between the boundary points. Satisfaction of the boundary condition on the check points is considered a measure of the accuracy of the simulation technique. The number of simulation charges *N* and *n* as well as the factors f_1 , β_1 , β_2 and β_3 are changed until satisfactory simulation accuracy is achieved.

B. Forces Acting on the Particle

When a particle is positioned between the discharge wire and the grounding collecting plate, it will be affected by the three forces [6] expressed as follows:

1. Van Der Waal's force F_{ν} ; which depends mainly on Hamaker constant H_c, the particle radius $r_{\rm p}$ and the particle distance from the collecting plate $Z_{\rm o}$. This force is expressed as:

$$F_{\nu} = \frac{2H_c r_p}{12Z_o^2} \tag{4}$$

2. Jonshon Rahbek force F_j ; which depends mainly on the particle radius r_p and the particle distance from the plate Z_0 . This force is expressed as:

$$F_j = Z_o^{\frac{3}{2}} \cdot n_z \tag{5-a}$$

$$n_z = \sqrt{\frac{16r_p}{9\pi^2(k_1 + k_2)^2}}$$
(5-b)

$$k_1 = \frac{1 - \nu_1^2}{\pi k_{\nu}}$$
(5-c)

$$k_2 = \frac{1 - \nu_2^2}{\pi k_y} \tag{5-d}$$

where v_1 and v_2 are Poisson ratio of particle and k_y is Young's modulus of particle.

3. Repulsion force F_1 ; which depends mainly on the particle radius r_p and the electric field. This force is expressed as:

$$F_1 = 0.832 \frac{2\pi^3}{3} \epsilon_o \epsilon_r r_p^2 E_n^2 \tag{6}$$

ii)

III. RESULTS AND DISCUSSION

The ESP discharge wire has 1 mm radius and is positioned at spacing H of 1 cm from the grounded collecting plates. The particle is positioned at height Z_0 in the range 0.2-0.6 nm from the grounded collecting plate. The particle is assumed spherical in shape with $r_{\rm p}$ varying in the range 0.1-2 µm in case of dielectric particles and in the range 0.1-5 µm in case of metal particles. The exhaust of diesel engine is loaded by carbon particles with relative permittivity varying in the range 2.6-18.8 [7, 8]. The value of the Poisson ratio v_1 and v_2 of the carbon are equal to 0.206. The value of the Young's modulus k_y of carbon is 1.565×10¹⁰ [9]. Number of wire simulation charges N is 45 for dielectric particles against 20 charges for metal particle. Number of particle charges n is 7 charges for the inner and outer charges for dielectric particles against 30 charges for metal particles. The fractions f_1 , β_1 , β_2 and β_3 values are 0.5, 0.5, 1.5 and 0.5, respectively. This results in satisfactory accuracy of the charge simulation method where the boundary conditions

are satisfied with an error not exceeding 1×10^{-7} %.

A. Electric Field Distribution

Fig. 3 (a) and (b) show the distribution of the electric field along the distance between the particle and the discharge wire as influenced by the distance Z_o at V = 22.5 kV. The figures show that the distribution of the electric field is not influenced by the distance Z_o for both metal and dielectric particles. This is because the distance between the particle and the discharge wire is much larger than Z_o . For metal particle, the electric field is high at the surface and decreases gradually in the dierction away from the particle, Fig. 3 (a). Then, the field starts to increase again in the dierction toward the discharge wire. This is in conformity to the fact that the field is inversly proportional to the curvature radius of the wire.

Fig. 4 (a) and (b) show the distribution of the electric field along the spacing between the particle and the grounded collecting plate as influenced by the distance Z_0 .



Fig. 3. Electric field distribution along the distance between the particle and the discharge wire as influenced by Z_{\circ} for (a) metal and (b) dielectric particles.



Fig. 4. Electric field distribution along the distance between the particle and the grounded plate as influenced by the distance Z_0 for (a) metal and (b) dielectric particles.

TABLE I Electric Field Values along the Distance $Z_{\scriptscriptstyle 0}$ in case of Metal Particle for Relative Position $Z_{\scriptscriptstyle 0}$ of 0.2 mm



Fig. 5. Forces $F_v + F_j$ and F_1 acting on the particle radius as influenced by the relative permittivity ε_r for (a) metal and (b) dielectric particles. epslon_r = ε_r .



Fig. 6. Forces F_v+F_j and F_1 acting on the particle radius as influenced by the distance for (a) metal and (b) dielectric particles of $\varepsilon_r = 2.6$.

The figures show that the electric field decreases with the increase of Z_0 for both metal and dielectric particles. As the distance Z_0 is very small, the electric field between the particle and the collecting plate is almost uniform as shown in Fig. 4 and Table I. The electric field values are larger for the metal particle when compared with those of the dielectric particle of the same radius r_p .

B. Forces Acting on Particle in Vicinity of Collecting Plates

On stressing the discharge wire by 22.5 kV, the particle is exposed to three forces; Van Der Waals, Johnsen Rahbek and repulsion forces. Changing the relative permittivity ϵ r affects the repulsion force as expressed in Eq. (6). Fig. 5 (a) and (b) show how the forces F_v+F_j and F_1 are influenced by particle radius r_p

and the relative permittivity ε_r in case of metal and dielectric particles. The figures show that the particle radius at which the re-entrainment takes place decreases with the increase of the relative permittivity for both metal and dielectric particle.

Re-entrainment takes place at particle radius of values 4.8, 1.8 and 1.2 μ m corresponding to respective ϵ_r values of 2.6, 9 and 18.8 for metal particle. Also, re-entrainment takes place at particle radius of 1.8 and 0.09 μ m corresponding to respective ϵ_r values of 2.6 and 18.8 for dielectric particles.

As stated in Eqs. (4) and (5-a), the Van Der Waal F_v and Johnsen Rahbek F_j forces are affected by the relative position of the particle with respect to the collecting plat as determined by the distance Z_o in Fig. 2. Fig. 6 (a) and (b) show that the particle radius at which the reentrainment takes place is influenced by the distance Z_o in case of metal and dielectric particles. The figures show that the particle radius at which the re-entrainment takes place increases with the increase of Z_0 for both metal and dielectric particles. Re-entrainment takes place at particle radius values 4.5, 4.8 and 6.9 µm corresponding to respective Z_0 values of 0.3, 0.4 and 0.6 nm for metal particles. Also, re-entrainment takes place at particle radius values of 1.6, 1.8 and 2.4 µm corresponding to respective Z_0 values of 0.2, 0.4 and 0.6 nm for dielectric particles.

In the all previous cases, it is noted that the particle radius at which the re-entrainment takes place in case of dielectric particle is always smaller than that of metal one. Also, it is noted that the re-entrainment takes place in case of dielectric particle at particles radius greater than 1 μ m in agreement with previous findings [1].

IV. CONCLUSION

- Among the forces acting on a particle nearby the collecting plate of an ESP is the repulsion force. This force depends on the electric field value, which is estimated using the charge simulation method.
- 2) Re-entrainment takes place when the repulsion force exceeds the resultant of both the Van Der Waal and Johnsen Rahbek forces.
- Re-entrainment takes place for dielectric particles at particle radius greater than 1 μm in agreement with previous findings.
- Particle radius values at re-entrainment for dielectric particles are smaller than those for metal ones.

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REFERENCES

- T. Yamamoto, T. Mimura, N. Otsuka, Y. Ito, Y. Ehara, and A. Zukeran, "Diesel PM Collection for Marine and Automobile Emissions Using EHD Electrostatic Precipitators," *IEEE Transactions on Industry Applications*, vol. 46, pp. 1606-1612, 2010.
- [2] A. Zukeran, Y. Ikeda, Y. Ehara, M. Matsuyama, T. Ito, T. Takahashi, H. Kawakami, and T. Takamatsu, "Two-stage-type electrostatic precipitator re-entrainment phenomena under diesel flue gases," *IEEE Transactions on Industry Applications*, vol. 35, pp. 346-351, 1999.
- [3] H. Singer, H. Steinbigler, and P. Weiss, "A Charge Simulation Method for the Calculation of High Voltage Fields," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-93, pp. 1660-1668, 1974.
- [4] M. Abdel-Salam, High-Voltage Engineering: Theory and Practice. New York, NY, Marcel Dekker, 2nd ed. ch. 2, 2000.
- [5] M. Abdel-Salam, "Discharge Inception in High-Voltage Tree-Electrodes System", *Journal of Physics D: Applied Physics*, vol. 20, pp. 629-634, 1987.

- [6] Y. Mochizuki, S. Sakakibara, and H. Asano, "Electrical Reentrainment of Particles Deposited on Collecting Plate in Electrostatic Precipitator", presented at the 8th International Conference on Electrostatic Precipitation, Birmingham, AL, USA, pp. 1-12, May 12-17, 2001.
- [7] https://www.vega.com/home_dk//media/PDFfiles/List_of_dielectr ic_constants_EN.ashx
- [8] http://www.clippercontrols.com/pages/Dielectric-Constant-Values.html
- [9] http://www.azom.com/properties.aspx?ArticleID=516

APPENDIX

Calculation of normal component of the electrical field

The electric field at a boundary point due to the simulation charges has three components; E_x -, E_y - and E_z - components. Based on these components, the normal component of the electric field E_n is determined.

A. Cartesian Components of Electrical Field

The expressions determining the electric field components depend on the type of the simulation charge.

A.1 Point Charge

The electric field components are expressed at boundary point whose coordinates (x_b , y_b , z_b) due to the *j*th point charge at ($x_c(j)$, $y_{cj}(j)$, $z_c(j)$) as [3]:

$$E_{\chi} = \sum_{j=1}^{N} \left[\frac{(x_b - x_c(j))}{(\sqrt{(x_b - x_c(j))^2 + (y_b - y_c(j))^2 + (z_b - z_c(j))^2})^3} - \frac{(x_b - x_c(j))}{(\sqrt{(x_b - x_c(j))^2 + (y_b + y_c(j))^2 + (z_b - z_c(j))^2})^3} \right] \cdot \frac{Q(j)}{4\pi\varepsilon_0}$$

$$E_{y} = \sum_{j=1}^{N} \left[\frac{(y_{b} - y_{c}(j))}{(\sqrt{(x_{b} - x_{c}(j))^{2} + (y_{b} - y_{c}(j))^{2} + (z_{b} - z_{c}(j))^{2}})^{3}} - \frac{(y_{b} + y_{c}(j))}{(\sqrt{(x_{b} - x_{c}(j))^{2} + (y_{b} + y_{c}(j))^{2} + (z_{b} - z_{c}(j))^{2}})^{3}} \right] \cdot \frac{Q(j)}{4\pi\varepsilon_{o}}$$

$$E_{z} = \sum_{j=1}^{N} \left[\frac{(z_{b} - z_{c}(j))}{(\sqrt{(x_{b} - x_{c}(j))^{2} + (y_{b} - y_{c}(j))^{2} + (z_{b} - z_{c}(j))^{2})^{3}}} - \frac{(z_{b} + z_{c}(j))}{(\sqrt{(x_{b} - x_{c}(j))^{2} + (y_{b} + y_{c}(j))^{2} + (z_{b} - z_{c}(j))^{2}})^{3}} \right] \cdot \frac{Q(j)}{4\pi\varepsilon_{o}}$$

(7-c)

(7-a)

The total electric field is expressed as:

$$E_t = \sqrt{(E_x^2 + E_y^2 + E_z^2)}$$
(7-d)

A.2 Infinite Line Charge

The electric field components are expressed at boundary point whose coordinates $(x_b, 0, z_b)$ due to the *j*th infinite line charge at $(x_i(j), 0, z_i(j))$ as [3]:

$$E_{x} = \sum_{j=1}^{N} \left[\frac{(x_{b} - x_{c}(j))}{((x_{b} - x_{c}(j))^{2} + (z_{b} - z_{c}(j))^{2})^{3}} - \frac{(x_{b} - x_{c}(j))}{((x_{b} - x_{c}(j))^{2} + (z_{b} + z_{c}(j))^{2})^{3}} \right] \cdot \frac{\lambda(j)}{4\pi\varepsilon_{o}}$$

$$(8-a)$$

$$E_{y} = 0 \tag{8-b}$$

$$E_{z} = \sum_{j=1}^{N} \left[\frac{(z_{b} - z_{c}(j))}{((x_{b} - x_{c}(j))^{2} + (z_{b} - z_{c}(j))^{2})^{3}} - \frac{(z_{b} - z_{c}(j))}{((x_{b} - x_{c}(j))^{2} + (z_{b} + z_{c}(j))^{2})^{3}} \right] \cdot \frac{\lambda(j)}{4\pi\varepsilon_{o}}$$

The total electric field is expressed as:

$$E_t = \sqrt{(E_x^2 + E_z^2)}$$
 (8-d)

(8-c)

A.3 Ring Charge

The radial component of the electric field is expressed at boundary point whose cylindrical coordinates (r_b , z_b) due to a ring charge of radius r_c at $z = z_c$ as [3]:

$$E_{r} = \frac{-Q}{4\pi\varepsilon_{o}} \cdot \frac{1}{\pi r} \left[\left(\frac{(r_{c}^{2} - r_{b}^{2} + (z_{b} - z_{c})^{2}) \cdot E(k_{1}) - \beta_{1}^{2} \cdot K(k_{1})}{\alpha_{1}\beta_{1}^{2}} \right) - \left(\frac{(r_{c}^{2} - r_{b}^{2} + (z_{b} + z_{c})^{2}) \cdot E(k_{2}) - \beta_{2}^{2} \cdot K(k_{2})}{\alpha_{2}\beta_{2}^{2}} \right) \right]$$

$$(9-a)$$

As shown in Fig. 7, the angle λ is the angle between the radial direction from the ring center and the *x*-axis in the *z*-plane of the ring.

Based on the angle λ , the E_{x} - and E_{y} -components of the electric field are expressed as:

$$E_x = E_r \cos \lambda \qquad (9-b)$$

$$E_y = E_r \sin \lambda \qquad (9-c)$$

The *z*-component of the electric field is expressed at boundary point whose cylindrical components (r_b , z_b) due to a ring charge of radius r_c at $z = z_c$ as:

$$E_z = \frac{-Q}{4\pi\varepsilon_o} \cdot \frac{2}{\pi} \cdot \left[\frac{(z_b - z_c)E(k_1)}{\alpha_1 \beta_1^2} + \frac{(z_b + z_c)E(k_2)}{\alpha_2 \beta_2^2} \right] \quad (9-d)$$

where:

$$\begin{aligned} \alpha_1 &= \sqrt{(x_b + x_c)^2 + (z_b - z_c)^2} \\ \alpha_2 &= \sqrt{(x_b + x_c)^2 + (z_b + z_c)^2} \\ \beta_1 &= \sqrt{(x_b - x_c)^2 + (z_b - z_c)^2} \\ \beta_2 &= \sqrt{(x_b - x_c)^2 + (z_b + z_c)^2} \end{aligned}$$



Fig. 7. Components E_x , E_y and E_r of the electric field due to a ring charge.

$$k_1 = \frac{2\sqrt{x_c x_b}}{\alpha_1}$$
$$k_2 = \frac{2\sqrt{x_c x_b}}{\alpha_2}$$

B. Normal Component of Electric Field



Fig. 8. Components E_x , E_z and E_n of the electric field at the particle surface.

The normal component of the electric field at a boundary point on the particle surface is calculated using the electric field components after being decomposed along the radial direction from the particle center as defined by the angle δ shown in Fig. 8. The angle δ determines the direction of the normal field component measured from the horizontal *x*-axis. Therefore, the angle δ is decided by the coordinates of the boundary point.